

Variable G Corrections to Statefinder Parameters of Dark Energy

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Abstract Motivated by several observational and theoretical developments concerning the variability of Newton’s gravitational constant with time $G(t)$, we calculate the varying G correction to the statefinder parameters for four models of dark energy namely interacting dark energy holographic dark energy, new-agegraphic dark energy and generalized Chaplygin gas.

Keywords Dark energy · Dark matter · Cosmological constant · Statefinder parameters

1 Introduction

Recent cosmological observations obtained by SNe Ia [1, 2], WMAP [3], SDSS [4] and X-ray [5] indicate that the observable universe experiences an accelerated expansion. Although the simplest way to explain this behavior is the consideration of a cosmological constant [6, 7], the known fine-tuning problem [8] led to the dark energy paradigm. The dynamical nature of dark energy, at least in an effective level, can originate from a variable cosmological “constant” [9, 10], or from various fields, such is a canonical scalar field (quintessence) [11–13], a phantom field, that is a scalar field with a negative sign of the kinetic term [14, 15], or the combination of quintessence and phantom in a unified model named quintom [16, 17]. Finally, an interesting attempt to probe the nature of dark energy according to some basic quantum gravitational principles is the holographic dark energy paradigm [18–20].

Dirac’s Large Numbers Hypothesis (LNH) is the origin of many theoretical studies of time-varying G . According to LNH, the value of \dot{G}/G should be approximately the Hubble rate [21]. Although it has become clear in recent decades that the Hubble rate is too high to be compatible with experiments, the enduring legacy of Dirac’s bold stroke is the acceptance by modern theories of non-zero values of \dot{G}/G as being potentially consistent with

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physical reality. Moreover, it is the task of a final quantum gravity theory to answer how, why and what a particular physical constant (like G) takes a specific value. As we are considering only variations of G with respect to temporal coordinate t , there is a suggestion to take $G(r)$, as a spatial (or radial) dependent quantity [22]. In [23], it is shown that G was actually varying before the electroweak era in the early Universe, while after the spontaneous symmetry breaking, the gravitational coupling G attained a finite constant value known today. This line of reasoning can be extended to other types of couplings in particle physics as well [24] but we shall not go into that. In the same manner, some authors [25] have also considered the speed of light as a time varying quantity.

There have been many proposals in the literature attempting to theoretically justified a varying gravitational constant, despite the lack of a full, underlying quantum gravity theory. Starting with the simple but pioneering work of Dirac [26], the varying behavior in Kaluza-Klein theory was associated with a scalar field appearing in the metric component corresponding to the 5-th dimension [27] and its size variation [28, 29]. An alternative approach arises from Brans-Dicke framework [30], where the gravitational constant is replaced by a scalar field coupling to gravity through a new parameter, and it has been generalized to various forms of scalar-tensor theories [31–33], leading to a considerably broader range of variable- G theories. In addition, justification of a varying Newton's constant has been established with the use of conformal invariance and its induced local transformations [34]. Finally, a varying G can arise perturbatively through a semiclassical treatment of Hilbert-Einstein action [35, 36], non-perturbatively through quantum-gravitational approaches within the "Hilbert-Einstein truncation" [37, 38], or through gravitational holography [39, 40].

There is convincing evidence of a varying Newton's constant G : Observations of Hulse-Taylor binary pulsar B1913 + 16 gives a following estimate $0 < \dot{G}/G \sim 2 \pm 4 \times 10^{-12} \text{ yr}^{-1}$ [41], helioseismological data gives the bound $0 < \dot{G}/G \sim 1.6 \times 10^{-12} \text{ yr}^{-1}$ [42] (see Ref. [43] for various bounds on \dot{G}/G from observational data). The variability in G results in the emission of gravitational waves. In another approach, it is shown that G can be oscillatory with time [44]. It is recently proposed that variable cosmic constants are coupled to each other i.e. variation in one leads to changes in others [45]. A variable gravitational constant also explains the dark matter problem as well [46]. Also discrepancies in the value of Hubble parameter can be removed with the consideration of variable G [47]. Motivated by the above arguments, we shall calculate the varying G corrections to the statefinder parameters for four chosen models of dark energy in the coming section and subsections.

2 The Model

The action of our model is [48]

$$S = \int d^4x \mathcal{L} = \int d^4x \left\{ \sqrt{-g} \left[\frac{R}{G} + F(G) \right] + \mathcal{L}_m \right\}, \quad (1)$$

where G is the Newton's gravitational constant and $F(G)$ is an arbitrary function of G while \mathcal{L}_m is a matter Lagrangian. From the Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial G} = \nabla_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu G)}, \quad (2)$$

to obtain

$$\frac{\partial F}{\partial G} = \frac{R}{G^2}. \tag{3}$$

Moreover from the variation with respect to $g_{\mu\nu}$, we have from (1),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} + g_{\mu\nu}\left(\frac{1}{2}GF(G)\right). \tag{4}$$

Writing

$$\frac{1}{2}GF(G) = \Lambda(t), \tag{5}$$

we get from (4)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} + g_{\mu\nu}\Lambda(t). \tag{6}$$

Sahni et al. [49] introduced a pair of cosmological diagnostic pair $\{r, s\}$ which they termed as Statefinder. The two parameters are dimensionless and are geometrical since they are derived from the cosmic scale factor alone, though one can rewrite them in terms of the parameters of dark energy and matter. Additionally, the pair gives information about dark energy in a model independent way i.e. it categorizes dark energy in the context of background geometry only which is not dependent on the theory of gravity. Hence geometrical variables are universal. Also this pair generalizes the well-known geometrical parameters like the Hubble parameter and the deceleration parameter. This pair is algebraically related to the equation of state of dark energy and its first time derivative.

The statefinder parameters were introduced to characterize primarily flat universe ($k = 0$) models with cold dark matter (dust) and dark energy. They were defined as

$$r \equiv \frac{\ddot{a}}{aH^3}, \tag{7}$$

$$s \equiv \frac{r - 1}{3(q - \frac{1}{2})}. \tag{8}$$

Here $q = -\frac{\ddot{a}}{aH^2}$ is the deceleration parameter.

For cosmological constant with a fixed equation of state ($w = -1$) and a fixed Newton’s gravitational constant, we have $\{1, 0\}$. Moreover $\{1, 1\}$ represents the standard cold dark matter model containing no radiation while Einstein static universe corresponds to $\{\infty, -\infty\}$ [50]. In literature, the diagnostic pair is analyzed for various dark energy candidates including holographic dark energy [51], agegraphic dark energy [52], quintessence [53], dilaton dark energy [54], Yang-Mills dark energy [55], viscous dark energy [56], interacting dark energy [63], tachyon [57], modified Chaplygin gas [58] and $f(R)$ gravity [59] to name a few.

For the present homogeneous, isotropic and spatially flat universe containing dark energy and dark matter (ignoring baryonic matter, radiation, neutrinos etc.), the Friedmann equation is

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_x). \tag{9}$$

Let us first consider dark energy obeying an equation of state of the form $p_x = w\rho_x$. The formalism of Sahni and coworkers [49] will be generalized to permit varying gravitational

constant. In this case, the definition of s is generalized to

$$s \equiv \frac{r - \Omega}{3(q - \frac{\Omega}{2})}. \quad (10)$$

Here $\Omega = \Omega_m + \Omega_x$. The deceleration parameter may be expressed as

$$q = \frac{1}{2}[\Omega_m + (1 + 3w)\Omega_x]. \quad (11)$$

Hence if Ω_x , Ω and q are determined by measurements the equation of state factor w may be found from

$$w = \frac{2q - \Omega}{3\Omega_x}. \quad (12)$$

Differentiation of parameter q , together with (7) leads to

$$r = 2q^2 + q - \frac{\dot{q}}{H}. \quad (13)$$

From (11), we have

$$\dot{q} = \frac{1}{2}\dot{\Omega}_m + \frac{1}{2}(1 + 3w)\dot{\Omega}_x + \frac{3}{2}\dot{w}\Omega_x. \quad (14)$$

Furthermore,

$$\dot{\Omega} = \frac{\dot{\rho}}{\rho_{\text{cr}}} - \frac{\rho}{\rho_{\text{cr}}^2}\dot{\rho}_{\text{cr}}, \quad (15)$$

with

$$\dot{\rho}_{\text{cr}} = \rho_{\text{cr}} \left(2\frac{\dot{H}}{H} - \frac{\dot{G}}{G} \right), \quad (16)$$

and

$$\dot{H} = -H^2(1 + q). \quad (17)$$

Hence

$$\dot{\rho}_{\text{cr}} = -H\rho_{\text{cr}}[2(1 + q) + \Delta_G], \quad (18)$$

where $\Delta_G \equiv G'/G$, $\dot{G} = HG'$ which leads to

$$\dot{\Omega} = \frac{\dot{\rho}}{\rho_{\text{cr}}} + \Omega H[2(1 + q) + \Delta_G]. \quad (19)$$

For cold dark matter $\dot{\rho}_m = -3H\rho_m$, (19) gives

$$\dot{\Omega}_m = \Omega_m H(-1 + 2q + \Delta_G), \quad (20)$$

and for dark energy $\dot{\rho}_x = -3(1 + w)H\rho_x$, (19) gives

$$\dot{\Omega}_x = \Omega_x H(-1 - 3w + 2q + \Delta_G). \quad (21)$$

Inserting (20) and (21) into (14) and the resulting expression into (13) finally leads to

$$r = \Omega_m + \left[1 + \frac{9}{2}w(1+w) \right] \Omega_x - \frac{3}{2H} \dot{w} \Omega_x - \frac{\Delta_G}{2} [\Omega_m + (1+3w)\Omega_x]. \tag{22}$$

Inserting the expression (22) into (10) leads to

$$s = 1 + w - \frac{\dot{w}}{3wH} - \frac{\Delta_G}{9w\Omega_x} [\Omega_m + (1+3w)\Omega_x]. \tag{23}$$

Assuming a spatially flat universe satisfying $\Omega_m = 1 - \Omega_x$, we obtain from (22) and (23), the following expressions

$$r = 1 + \frac{9}{2}w(1+w)\Omega_x - \frac{3}{2H} \dot{w} \Omega_x - \frac{\Delta_G}{2}(1+3w\Omega_x), \tag{24}$$

$$s = 1 + w - \frac{\dot{w}}{3wH} - \frac{\Delta_G}{9w\Omega_x}(1+3w\Omega_x). \tag{25}$$

Note that (24) and (25) provide necessary variable G corrections to statefinder parameters of Sahni et al. [49]. In particular for the Λ CDM model, the above pair gives

$$r = -\frac{\Delta_G}{2}(1-3\Omega_x), \tag{26}$$

$$s = \frac{\Delta_G}{9\Omega_x}(1-3\Omega_x). \tag{27}$$

Moreover the standard cold dark matter corresponds to $\{-\Delta_G/2, -\infty\}$. For a universe containing only dark energy i.e. $\Omega_m = 0$, we get from (22) and (23):

$$r = \left[1 + \frac{9}{2}w(1+w) \right] \Omega_x - \frac{3}{2H} \dot{w} \Omega_x - \frac{\Delta_G}{2}(1+3w)\Omega_x, \tag{28}$$

$$s = 1 + w - \frac{\dot{w}}{3wH} - \frac{\Delta_G}{9w}(1+3w). \tag{29}$$

2.1 Interacting Dark Energy

There is a class of cosmological models in which evolution of the universe depends on the interaction of cosmic components like dark energy and dark matter (and possibly radiation and neutrinos). Models in which the main energy components do not evolve separately but interact with each other bear a special interest since they may alleviate or even solve the “cosmic coincidence problem”. The problem can be summarily stated as “why now?”, that is to say: “Why the energy densities of the two main components happen to be of the same order today?”

The conservation equations for the interacting dark energy-dark matter are

$$\begin{aligned} \dot{\rho}_x + 3H\rho_x(1+w) &= -Q, \\ \dot{\rho}_m + 3H\rho_m &= Q. \end{aligned} \tag{30}$$

The coupling between dark components could be a major issue to be confronted in studying the physics of dark energy. However, so long as the nature of these two components remain

unknown it will not be possible to derive the precise form of the interaction from first principles. Therefore, one has to assume a specific coupling from the outset [60–62] or determine it from phenomenological requirements [63]. Here we take $Q = -3\Pi H$ which measures the strength of the interaction [63]. For later convenience we will write it as $Q = -3\Pi H$ where the new quantity Π has the dimension of a pressure.

The second Friedmann equation is

$$\dot{H} = -4\pi G(\rho_t + p), \tag{31}$$

where $\rho_t = \rho_m + \rho_x$. Notice that if the cosmic energy density and pressure violate the null energy condition $\rho_t + p < 0$, then $\dot{H} > 0$, i.e. the Hubble horizon H^{-1} will be shrinking. This situation can arise in a phantom-type dark energy model. Also if $\rho_t + p > 0$ then the Hubble horizon is expanding and this can be due to a quintessence-like dark energy. Moreover for a cosmological constant dominated universe ($\rho_t + p = 0$) or during inflation in the early universe, the Hubble horizon is a fixed quantity. Differentiating (31) w.r.t. t and using (30) and (9), we get

$$\frac{\ddot{H}}{H^3} = \frac{9}{2} \left(1 + \frac{p}{\rho_t} \right) + \frac{9}{2} \left[w(1+w) \frac{\rho_x}{\rho_t} - \frac{w\Pi}{\rho_t} - \frac{\dot{w}\rho_x}{3H\rho_t} \right] - \frac{3}{2} \Delta_G \left(1 + \frac{p}{\rho_t} \right). \tag{32}$$

At variance with H and \dot{H} , the second derivative \ddot{H} does depend on the interaction between components. Consequently, to discriminate between models with different interactions or between interacting and non-interacting models, it is desirable to characterize the cosmological dynamics additionally by parameters that depend on \ddot{H} .

Consequently r can be written in alternative form as

$$r = \frac{\ddot{H}}{H^3} - 3q + 2. \tag{33}$$

Using (32) in (33) and the expression of $q = -\frac{1}{2} - \frac{3}{2} w\Omega_x (= -\frac{1}{2} - \frac{3p}{2\rho_t})$, we obtain

$$r = 1 + \frac{9}{2} \frac{w}{1+\kappa} \left(1 + w - \frac{\Pi}{\rho_x} - \frac{\dot{w}}{3Hw} \right) - \frac{3}{2} \Delta_G \left(1 + \frac{w}{1+\kappa} \right), \tag{34}$$

where $\kappa \equiv \rho_m/\rho_x$ is a dimensionless but a varying quantity. Note that a constant κ require extreme fine-tuning. Moreover for a fixed κ , the cosmic coincidence problem is trivially resolved since the universe will remain in that state forever. To resolve the coincidence problem, the density ratio κ has to be slowly varying over a time of the order H^{-1} .

Making use of (34) in (8), we obtain

$$s = 1 + w - \frac{\Pi}{\rho_x} - \frac{\dot{w}}{3Hw} - \frac{\Delta_G}{3w} (1 + \kappa + w). \tag{35}$$

Note that for $\Delta_G = 0$, the above expressions (34) and (35) reduce to the expressions studied in [63], therefore (34) and (35) provide necessary variable G corrections to the statefinder parameters for an interacting dark energy model.

In a paper [64], the authors showed that scaling solutions of the form $\kappa \sim a^{-\xi}$, where ξ denotes a constant parameter in the range $[0, 3]$ can be obtained when the dark energy component decays into the pressureless matter. These solutions are interesting because they alleviate the coincidence problem [65–67]. Indeed a model with $\xi = 3$ amounts to the Λ CDM

model with $w = -1$ and $\Pi = 0$. As mentioned earlier, the $\xi = 0$ trivially resolves the coincidence problem, which is of no interest. Any solution $\xi < 3$ renders the coincidence problem less acute. In that scheme, with $w = \text{constant}$, it has been shown in [64] that the interactions which produce scaling solutions are given by

$$\frac{\Pi}{\rho_x} = \left(w + \frac{\xi}{3} \right) \frac{\kappa_0(1+z)^\xi}{1 + \kappa_0(1+z)^\xi}, \tag{36}$$

where $\kappa = \kappa_0(1+z)^\xi$, $z = (a_0/a) - 1$ is the redshift and $\kappa_0 \equiv \rho_{m0}/\rho_{x0}$ is the present density ratio. Putting (36) in (34) and (35), we get

$$r = 1 + \frac{9}{2} \frac{w}{1 + \kappa_0(1+z)^\xi} \left[1 + w - \left(w + \frac{\xi}{3} \right) \frac{\kappa_0(1+z)^\xi}{1 + \kappa_0(1+z)^\xi} \right] - \frac{3}{2} \Delta_G \left[1 + \frac{w}{1 + \kappa_0(1+z)^\xi} \right], \tag{37}$$

$$s = 1 + w - \left(w + \frac{\xi}{3} \right) \frac{\kappa_0(1+z)^\xi}{1 + \kappa_0(1+z)^\xi} - \frac{\Delta_G}{3w} [1 + w + \kappa_0(1+z)^\xi]. \tag{38}$$

2.2 Holographic Dark Energy

The holographic dark energy is constructed in the light of the holographic principle. Its framework is the black hole thermodynamics [68, 69] and the connection (known from AdS/CFT correspondence) of the UV cut-off of a quantum field theory, which gives rise to the vacuum energy, with the largest distance of the theory [70–77]. Thus, determining an appropriate quantity L to serve as an IR cut-off, imposing the constraint that the total vacuum energy in the corresponding maximum volume must not be greater than the mass of a black hole of the same size, and saturating the inequality, one identifies the acquired vacuum energy as holographic dark energy:

$$\rho_x = \frac{3c^2}{8\pi G} \frac{1}{L^2}. \tag{39}$$

Here c is the holographic parameter of order unity. Note that the IR cut-off L can be of several types such as particle horizon, Hubble horizon and future event horizon. The definition of holographic dark energy is sensitive to the choice of each horizon and gives different dynamical features corresponding to each horizon. For the present study, we don't need to choose a specific length scale and work with general L .

The time evolution of (39) is

$$\dot{\rho}_x = -\rho_x H \left(2 - \frac{2}{c} \sqrt{\Omega_x} + \Delta_G \right). \tag{40}$$

Using (40) in the energy conservation equation yields

$$w = \frac{1}{3} \left(-1 - \frac{2}{c} \sqrt{\Omega_x} + \Delta_G \right). \tag{41}$$

Differentiating (41), we obtain

$$\frac{1}{H} \dot{w} \equiv \frac{dw}{d \ln a} = -\frac{1}{3c} \sqrt{\Omega_x} (1 - \Omega_x) \left(1 + \frac{2}{c} \sqrt{\Omega_x} - \Delta_G \right) + \frac{1}{3} \Delta'_G, \tag{42}$$

where we have used

$$\dot{\Omega}_x = H\Omega_x(1 - \Omega_x)\left(1 + \frac{2}{c}\sqrt{\Omega_x} - \Delta_G\right). \tag{43}$$

Putting (42) and (43) in (24) and (25), we obtain

$$\begin{aligned} r = & 1 + \Omega_x\left(-1 - \frac{2}{c}\sqrt{\Omega_x} + \Delta_G\right)\left(1 - \frac{2}{c}\sqrt{\Omega_x} + \frac{3}{2}\Delta_G\right) \\ & + \frac{1}{2c}\Omega_x^{3/2}(1 - \Omega_x)\left(1 + \frac{2}{c}\sqrt{\Omega_x} - \Delta_G\right) \\ & + \frac{1}{3}\Delta'_G\Omega_x - \frac{\Delta_G}{2}\left[1 + \Omega_x\left(-1 - \frac{2}{c}\sqrt{\Omega_x} + \Delta_G\right)\right]. \end{aligned} \tag{44}$$

$$\begin{aligned} s = & \frac{2}{3} - \frac{\sqrt{\Omega_x}}{c} - \frac{\Omega_x^{3/2}}{3c} + \frac{\Delta_G}{3} + \frac{\Delta'_G}{3}\left(1 + \frac{2}{c}\sqrt{\Omega_x} - \Delta_G\right)^{-1} + \frac{\Delta_G}{3\Omega_x}\left(1 + \frac{2}{c}\sqrt{\Omega_x} - \Delta_G\right)^{-1} \\ & \times \left[1 + \Omega_x\left(-1 - \frac{2}{c}\sqrt{\Omega_x} + \Delta_G\right)\right]. \end{aligned} \tag{45}$$

2.3 New-Ageographic Dark Energy

An interesting attempt for probing the nature of dark energy (DE) is the so-called “agegraphic DE” (ADE). This model was recently proposed [78] to explain the acceleration of the universe expansion within the framework of a fundamental theory such as quantum gravity. The ADE model assumes that the observed DE comes from the spacetime and matter field fluctuations in the universe. Following the line of quantum fluctuations of spacetime, Karolyhazy et al. [79] discussed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \beta t_p^{2/3} t^{1/3}$ where β is a dimensionless constant of order unity. Based on Karolyhazy relation and Maziashvili arguments [80, 81], Cai proposed the original ADE model to explain the acceleration of the universe expansion [78]. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, a new model of ADE was proposed by Wei and Cai [82], while the time scale was chosen to be the conformal time η instead of the age of the universe.

The definition of NADE is given by [82]

$$\rho_x = \frac{3n^2}{8\pi G} \frac{1}{\eta^2}, \tag{46}$$

where n is a constant of order unity. Its time evolution is

$$\dot{\rho}_x = -H\rho_x\left(\frac{2}{3na}\sqrt{\Omega_x} + \Delta_G\right). \tag{47}$$

Making use of (47) in the energy conservation equation yields

$$w = -1 + \frac{2}{3na}\sqrt{\Omega_x} + \frac{\Delta_G}{3}. \tag{48}$$

Differentiating (48) w.r.t. t gives

$$\frac{1}{H}\dot{w} = \frac{1}{3na}\sqrt{\Omega_x}(1 - \Omega_x)\left(3 - \frac{2}{3na}\sqrt{\Omega_x} - \frac{\Delta_G}{3}\right) - \frac{2}{3na}\sqrt{\Omega_x} + \frac{1}{3}\Delta'_G, \tag{49}$$

where we have used

$$\dot{\Omega}_x = H\Omega_x(1 - \Omega_x)\left(3 - \frac{2}{na}\sqrt{\Omega_x} - \Delta_G\right). \tag{50}$$

Using (48) and (49) in (24) and (25), we get

$$\begin{aligned} r = & 1 + \frac{9}{2}\Omega_x\left(\frac{2}{3na}\sqrt{\Omega_x} + \frac{\Delta_G}{3}\right)\left(-1 + \frac{2}{3na}\sqrt{\Omega_x} + \frac{\Delta_G}{3}\right) \\ & - \frac{1}{2na}\Omega_x^{3/2}(1 - \Omega_x)\left(3 - \frac{2}{3na}\sqrt{\Omega_x} - \frac{\Delta_G}{3}\right) \\ & + \frac{1}{na}\Omega_x^{3/2} - \frac{1}{2}\Omega_x\Delta'_G - \frac{\Delta_G}{2}\left[1 + \Omega_x(\Delta_G - 3) + \frac{2}{na}\Omega_x^{3/2}\right]. \end{aligned} \tag{51}$$

$$\begin{aligned} s = & \frac{2}{3na}\sqrt{\Omega_x} + \frac{\Delta_G}{3} - \frac{1}{3}\left(-1 + \frac{2}{3na}\sqrt{\Omega_x} + \frac{\Delta_G}{3}\right)^{-1} \\ & \times \left[\frac{1}{3na}\sqrt{\Omega_x}(1 - \Omega_x)\left(3 - \frac{2}{3na}\sqrt{\Omega_x} - \frac{\Delta_G}{3}\right) - \frac{2}{3na}\sqrt{\Omega_x} + \frac{1}{3}\Delta'_G\right] \\ & - \frac{\Delta_G}{3\Omega_x(\Delta_G - 3) + \frac{6}{na}\Omega_x^{3/2}}\left[1 + \Omega_x(\Delta_G - 3) + \frac{2}{na}\Omega_x^{3/2}\right]. \end{aligned} \tag{52}$$

2.4 Generalized Chaplygin Gas

Interest in generalized Chaplygin gas (GCG) arose when it appeared that it gives a unified picture of dark energy and dark matter. It solely provides the density evolution of matter at high redshifts and dark energy at low redshifts [83–85]. Other successes of GCG is that it explains the recent phantom divide crossing [86], is consistent with the data of type Ia supernova [87–89] and the cosmic microwave background [90–92]. The GCG emerges as an effective fluid associated with d -branes [93, 94] and can also be obtained from the Born-Infeld action [95]. In the present context, we are treating GCG as a dark energy candidate.

The equation of state for generalized Chaplygin gas is

$$p_x = -\frac{A}{\rho_x^\alpha}, \tag{53}$$

where α and A are constants. Using (53) in the energy conservation equation yields

$$\rho_x = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}. \tag{54}$$

Here B is an integration constant. The corresponding state parameter takes the form

$$w = -A\left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{-1}, \tag{55}$$

whose time derivative yields

$$\dot{w} = -3ABH(1 + \alpha) \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{-2}. \quad (56)$$

Using (55) and (56) in (24) and (25), we get

$$r = 1 + \frac{9}{2} AB\Omega_x (1 - a^{-3(1+\alpha)}) \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{-2} - \frac{\Delta_G}{2} \left[1 - 3A\Omega_x \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{-1} \right]. \quad (57)$$

$$s = 1 - (A + B(1 + \alpha)) \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{-1} + \frac{\Delta_G}{9A\Omega_x} \left(A + \frac{B}{a^{3(1+\alpha)}} \right) \times \left[1 - 3\Omega_x A \left(A + \frac{B}{a^{3(1+\alpha)}} \right)^{-1} \right]. \quad (58)$$

3 Concluding Remarks

In this paper, we calculated the corrections to statefinder parameters due to variable gravitational constant. These corrections are relevant because several astronomical observations provide constraints on the variability of G . An important thing to note is that the G -corrected statefinder parameters are still geometrical since the parameter Δ_G is a pure number and is independent of the geometry.

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